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Interactive exploration of a chaotic oscillator for generating musical signals in real-time concert performance.

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Abstract

A chaotic oscillator has a broad range of state space from which musicians can induce a variety of sounds. An application of such an oscillator for music performance and composition opens a door to a series of new projects in the music community. Chua's oscillator has been explored for musical signal generation in the context of music performance and composition. Further, an exploration of the parameter space of the oscillator has been assisted with evaluations of associated sound outputs guided by auditory perception. The application of Chua's oscillator in a music performance system requires several stages of research: 1) understanding basic principles of the chaotic system in terms of mathematical descriptions of the system and hardware configurations, 2) understanding the output signals in terms of vector space descriptions as well as understanding the auditory signal outputs in relation to the vector space descriptions, and 3) incorporating the system in a musical performance context. While the abundant papers available support the first stage of research, the second stage of research requires alternative ways of listening to the auditory signals with respect to the chaotic system behaviors. For the third stage methods and tools have been invented for exploration of the chaotic system, a simulation of the system in computer was implemented, and a peripheral performance system has been configured. In order to nourish auditory perception an efficient and intuitive way of interacting with the oscillator is essential. One of the most interesting preparations for this research was the design of an interactive graphical software interface, the *manifold interface.* This interface was extensively used for exploring parameter regions for pre-compositional activity and for sending control signals to both analog and simulated versions of the oscillator in real-time performance.

Introduction

Musical signals are acoustic signals produced by well-trained musicians who have a fine sense of tuning their physical movements while interacting with musical instruments, and an ability to achieve desired quality of sounds. Musical signals are often complex in their vibration patterns and the degree of this complexity is associated with the complexity arising in a listener's auditory percept. At the same time an intended refinement alternating between certain degrees of orderliness and transient qualities of sounds conceived by a performer, guides listeners to distinguish musical signals from other unintended noises. In contemporary use of the term, certain noises or unpleasing sounds can be brought into the definition of musical signals as long as their presence is contexualized with compositional decisions so that the listeners describe their experiences with those sounds as "musically meaningful." Thus when we speak of musical signals applying electronic and digital technology we are concerned with the variety of tone qualities which can be described in terms of complexity. The description, *complexity* may be based upon the analysis of the acoustic signals and upon a diagnosis of the computational cost to synthesize those acoustic signals in order to simulate a perceptually compatible tone quality. Through computer music history dating from 1956 one of the most non-trivial questions has been how to achieve with digital technology this variety of complexity that we experience from musical instruments.

In the computer music field, synthesis techniques such as additive synthesis [19], frequency modulation [6], linear predictive coding [13], and coupled resonator-filter models [20, 28] are applied for achieving the complex quality in synthesized sound. These techniques are helpful with one common drawback that an application of these techniques becomes more cumbersome or computationally expensive as the desired acoustic complexity increases. A chaotic oscillator such as Chua's oscillator suggests an alternative research direction and perspective since it is a paradigm for exploring a physical and mathematical system and the range of the oscillator's state space allows us to induce a variety of complex waveforms without having to assemble many waveform generators as is done for example in additive synthesis.

The application of the Chua's oscillator to sound synthesis has been recently studied in [1, 3, 21, 22, 23, 24, 15, 16]. In the application of continuous chaotic systems, two approaches are possible and may be considered complementary. One approach is to apply the chaotic system as one component of a simulation of physical instruments. In this approach nonlinearity in chaotic systems is used as a source for generating excitation patterns which are coupled to models of linear delay lines that describe tubes or strings of physical instruments. In these methods the nonlinear characteristics are usually held constant and the parameters associated with the linear delay line are varied during sound production. Methods to find the stable solutions for obtaining the control necessary to support this application in real-time performance have been discussed in [24]. Another approach is to apply the chaotic system as a signal generator and explore the circuit for its potential musical properties by varying the linear and nonlinear characteristics of the chaotic system itself. This paper presents studies from the second approach and introduces the reader to technical information concerning how to set up experimental and performance environments for musical signal generation with a continuous chaotic system.

1. Chaos and Sounds from the Chua's Circuit

Chua's circuit is an autonomous dynamical system that generates continuous signals. It is also chaotic fulfilling the working definition for chaotic systems [27, 31, 32]. A dynamical system can be called a chaotic system when it displays several fundamental features unique to the presence of chaos. One of those features can be characterized by a *mixing mechanism*, *stretching and folding*: a unit interval is stretched apart repeatedly by exponential growth until it reaches the ergodic limit of the system; then the two initial points are folded back together. While the stretching operation tends to pull apart nearby orbits, the folding operation brings them back together within a bounded region of phase space.[1] In addition to the mixing mechanism, stretching and folding, a chaotic system displays an asymptotic motion that is not an equilibrium point, periodic, or quasi-periodic, and this motion is often called chaotic motion. Further this chaotic motion is bounded to an asymptotic solution that possesses sensitive dependence on initial condition [29, 31]. For the readers who are interested, the formal mathematical proofs using the method called the Shil'nikov theorem for the presence of chaos in Chua's circuit can be found in the literature [26].

When a chaotic oscillator is applied to sound synthesis, the mixing mechanism produces frequencies of different periods in the acoustic signal. Frequencies accumulate in a listener's perception creating an experience of a complex tone. In the traditional analog sound synthesis studio we add the output of multiple waveform generators to arrive at the complexity of the signal we wish to construct. This is also the fundamental technique for sound synthesis in computer music. As an alternative system a single Chua's oscillator is capable of generating a range of signals from simple periodic signals to chaotic signals (see figure 1 and figure 2).

[1] Phase space is a collection of all possible states of a dynamical system.

Chaotic attractors contribute harmonic and inharmonic spectra and noise components to signals generated by a system. Signals from chaotic systems can be described in terms of stability and instability, patterns and their degrees of intermittency, transient qualities, and ambiguity of certain states, which amount to the complexity arising in our perception.

Figures 1a and 1b are Discrete Fourier Transforms of signals taken from the digital simulation of Chua's oscillator. Figure 1a shows the energy distribution of the signal relatively concentrated at a regular interval along the frequency spectra whereas figure 1b shows the energy distribution with somewhat unpredictable organization.

Figure 1a.

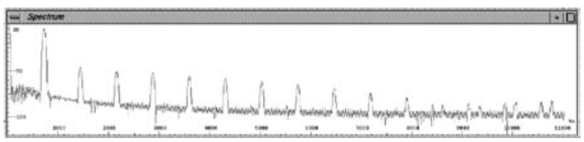


Figure 1b.

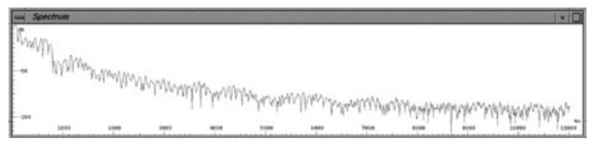


Figure 2a.

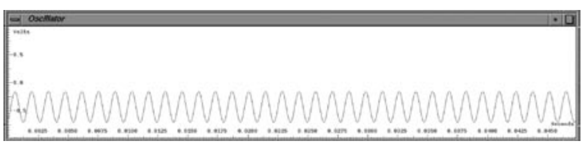
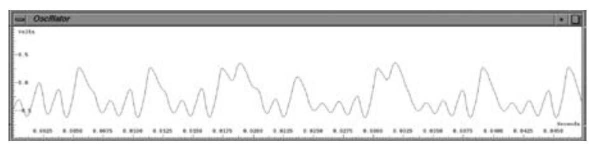


Figure 2b.



Figures 2a and 2b are the correspondent time-domain waveform representations of the signals in figures 1a and 1b. The signal in figure 1a is known as a limit cycle with period one; it's acoustic characteristic can be described as a stable tone with energy peaks in the frequency spectrum that reinforce our perception of the period-one frequency. Spectral energy peaks are referred to as partials; when the partials occur at regular intervals of near-integer ratios as shown in figure 1a they are said to be in harmonic relation. The signal in figure 1b is complex and aperiodic; our visual perception cannot determine well-defined frequency regions or patterns and the change of the waveform of the signal is unpredictable over time. This signal is known as a chaotic signal. The acoustic characteristic of a chaotic signal is noise-like, as our ear cannot settle into well-defined frequency regions or relationships among energy peaks in the spectral domain. The noise-like quality of the chaotic signal is due to chaotic distribution of the energy of the signal along the frequency spectra at all times.

The variety of signal characteristics generated from Chua's circuit are consequences of internal states of the system; the relationship between the control parameter setting and the emergence of possible states of the system is non-trivial. Experimental studies can be conducted by varying components of the system and observing how different combinations of control parameter values influence the state of the system. The technique for influencing the state of the system by varying components of the system, we will refer to as *parameter variation technique* [17] (see section 4.1).

2. Chua's Circuit: Physical Properties, Nonlinearity and Numerical Simulation

2.1. Physical properties of Chua's circuit

Chua's circuit is an electronic circuit which is designed to produce chaotic behavior in a physical system. It is made up of the minimum number of components that a circuit requires in order to demonstrate chaotic behavior [11]:

1) one locally active resistor,
2) three energy storage elements,
3) a non-linear element.

The basic elements of Chua's circuit include four linear circuit elements and a nonlinear resistor N_r which is called the *Chua's diode* [11]. The four linear elements consist of an inductor L, two capacitors C_1 and C_2, and a resistor R; a linear resistor R_0 is added in series to L (see Figure 3) in order to explicitly model the resistance characteristics of L. Chua's diode has a piecewise-linear driving point characteristic (DP) including a negative resistance that allows energy to be introduced into the system. The piecewise-linearity in N_r includes three segments with negative slopes; increasing the incline of the negative slopes describes increasing the amount of energy introduced into the circuit.

Figure 3.

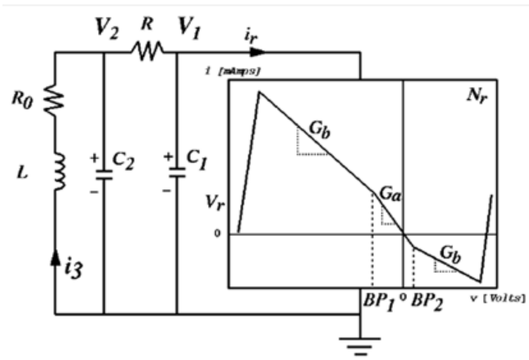


Figure 4.

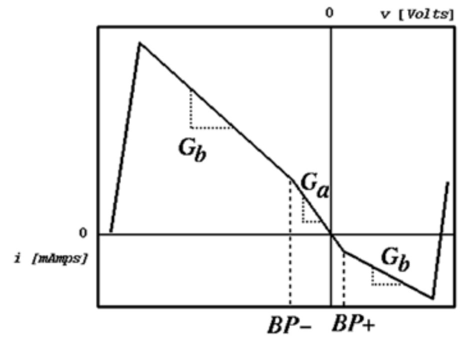


Figure 4 shows the five-segment piecewise-linear DP characteristic of N_r. The three center segments with negative resistance are referred to as the "locally active" region of N_r. The exterior segments with positive resistance indicate the circuit is "eventually passive," meaning these regions will dissipate energy no matter how much energy is introduced from an external source. This diode characteristic can be measured experimentally in physical circuits; in circuit simulations this characteristic is explicitly modeled to prevent the simulated signal from "blowing up" (approaching infinity) in these regions.

The boundaries between positive and negative resistance in the *v-i* characteristic are convergent regions for attractors. Signals approaching these regions can become caught by the Large Limit Cycle (LLC), a period-one oscillation producing an invariant and uninteresting sine-like tone. By observing the extreme phase positions of attractor trajectories it is possible to adjust the circuit parameters to maintain the steady-state solutions of attractors entirely in the negative resistance regions of the diode. This is important for maintaining the production of tones that exhibit chaotic properties and avoiding the LLC.

Chua's circuit can be constructed from off-the-shelf standard components; its implementation is approachable by following step by step procedures which can be found in various literature. An optimal way of implementing the circuit has been comprehensively presented in [12]. The nonlinear resistor N_r uses two op amps (operational amplifiers) and six linear resistors; for a diagram of physical components of N_r see figure 18. In a standard implementation the power supplies to the op amps are 9 volt batteries. The breakpoints of the piecewise-linear segments of N_r (see BP_1 and BP_2 in Figure 4) are proportional to the saturation levels of the op amps, which can be varied by varying the voltages of the power supplies. Therefore these voltages can be varied to adjust the symmetry of the breakpoint characteristics of N_r, which in turn affects the shape of the Chua's attractor and the harmonic content of the signal [22].

2.2. State equations for Chua's circuit

The system dynamics of the physical model of Chua's circuit can be described by a system of three ordinary differential equations referred to as the *global unfolding* of Chua's circuit [8]:

$$\left.\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C_1}\left[G(v_2 - v_1) - f(v_1)\right] \\ \frac{dv_2}{dt} &= \frac{1}{C_2}\left[G(v_1 - v_2) + i_3\right] \\ \frac{di_3}{dt} &= -\frac{1}{L}(v_2 + R_0 i_3) \end{aligned}\right\} \qquad (1)$$

where

$$G = \frac{1}{R}$$

These equations account for three parallel signal paths, the voltages crossing two capacitors (C_1, C_2), and the current flowing through the inductor (L). They are denoted by v_1, v_2, and i_3 in equation 1 and 2. These make a three-dimensional signal, which is required for a continuous system to exhibit chaotic behavior [11]. Another requirement is a nonlinearity, provided by the function $f(v_1)$ given in Figure 4, defined by:

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)\{|v_1 + E| - |v_1 - E|\} \qquad (2)$$

"Unfolding" refers to the theory in mathematics of adding parameters to dynamical equations - in this case R_0 added in the third differential equation - in order to generalize their dynamical behavior [8]. In the case of Chua's circuit these equations are defined over the entire state space of the circuit signal, making them "global." This circuit has been referred to as the *Chua's* *oscillator* [14], depicted in figure 3. It is canonical[2] with trajectories from a large class of three-piecewise non-linear electronic circuits. For efficiency BP_1 and BP_2 are described as a symmetrical pair; in practice this symmetry may be varied; in fact the option to vary the relationship between BP_1 and BP_2 from symmetrical to asymmetrical is important to achieve timbre control in tone production.

2.3. Global behavior of signals from Chua's oscillator

Each of the three energy storage elements in the Chua's circuit produces oscillations that can be converted into audio signals. The behavior of the voltage across these three elements is sensitive to the state of the circuit. The elements share a common operating frequency, and changes made anywhere in the circuit affect all three oscillations. These may be studied as a signal conveying three-dimensional information. Given an initial condition, an ODE of n-dimensions describes a *state vector* or *vector field* of the same number of dimensions. This solution is called a *trajectory*; it describes the direction and speed of the signal in time steps. Acoustic properties from the circuit depend upon the solution of the ODE for every dimension. The number of energy storage elements, their voltages, the shape and slope of the *v-i* characteristic of the nonlinear resistor, are all encoded in the acoustic signal.

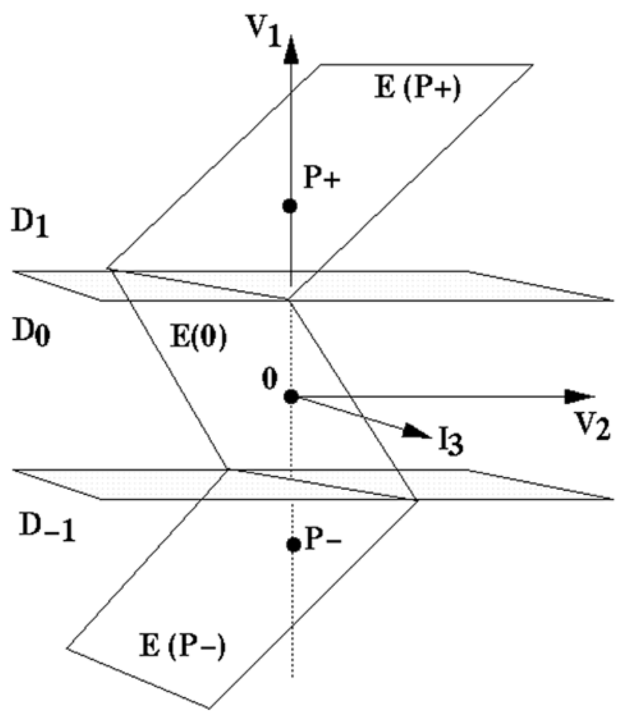
The *v-i* characteristic of the Chua's diode creates boundaries in the vector field. This can be seen in figures 4 and 5. The breakpoints BP_1 and BP_2 in figure 4 divide the state space in figure 5 into three regions, D_0, D_{-1} and D_1. This is only a qualitative schematic picture; it is not drawn for a specific set of parameters.

When the chaotic trajectory lies within only two of the three regions, D_{-1} and D_0 or D_1 and D_0, we call the attractor a *Spiral Chua's attractor*. Figure 6 shows a stable period-4 spiral attractor; this attractor bifurcates into a chaotic attractor. Strong harmonic components as well as deterministic noise can be found in spiral chaotic attractors; the expanding orbits of the spiral creates a series of tones of increasing amplitude. When an orbit stabilizes in a *n*-periodic attractor the longest orbit of the limit cycle becomes the lowest tone in our auditory perception

[2] "Canonical" means that the circuit trajectories in vector space are topologically conjugate ("qualitatively equivalent") with a large class of vector fields generated by 3-D circuits containing a piecewise-linear nonlinearity[7].

of the attractor (see figure 7) and this lowest tone functions as a base or fundamental supporting *n-1* number of tones above. A well-established lowest tone contributes the perceptual effect of stabilization to the tone quality of the attractor. In chaotic attractors the length of the fundamental varies unpredictably and its orbits span in broad bands creating unfocused auditory percept as well as an absense of any stabilized tone perception.

Figure 5.



As resistance is reduced the trajectories from D_1 or D_{-1} flow continuously into each other. When the trajectory crosses the middle (D_0) region and visits both outer regions, we call the attractor a *Double-Scroll Chua's attractor*. Figure 8 shows a chaotic double-scroll attractor; stable periodic attractors are also found in the region of the double scroll attractor for various parameter values. The balance of noise and pitch varies with the stability of the period of the attractor (see also section 4.3.2). A Double-Scroll Chua's attractor is a single compound attractor reflecting a mirror image from 0 to P+ and to P-. In the phase portrait of this attractor, the symmetry of the nonlinearity is clearly reflected in the shape of the portrait. Using parameter variation technique, as the symmetry of BP_1 to BP_2 is varied to asymmetry, one can observe the size of an orbit collapsing on one side, eventually creating a spiral attractor.

Figure 6.

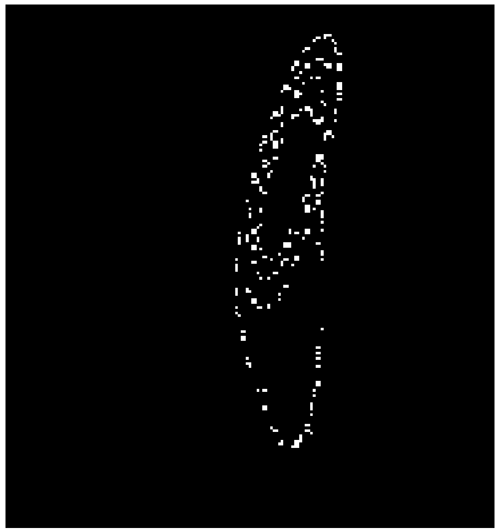


Figure 7b.

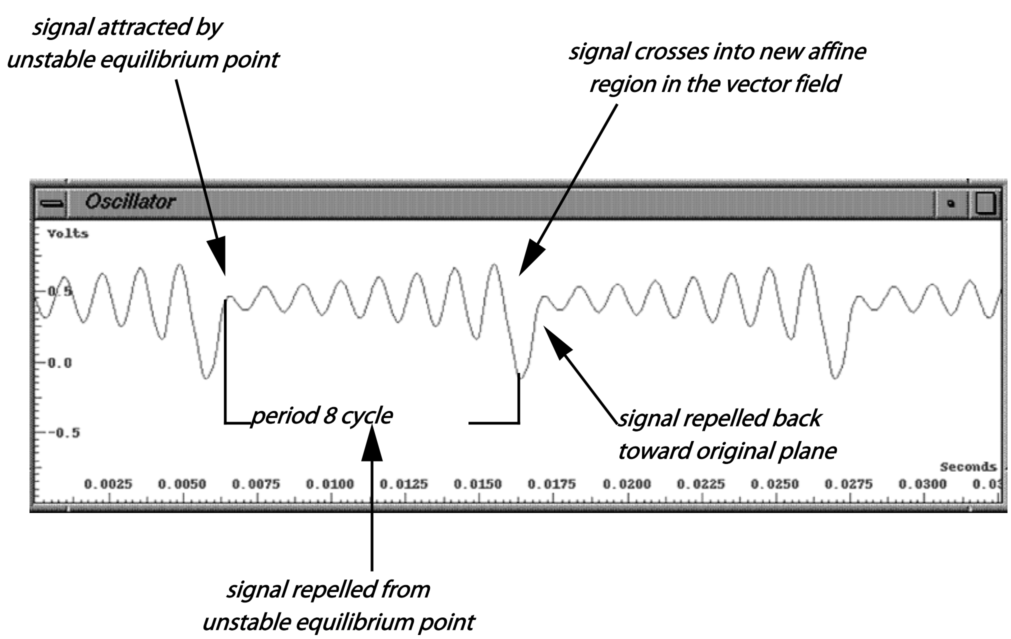
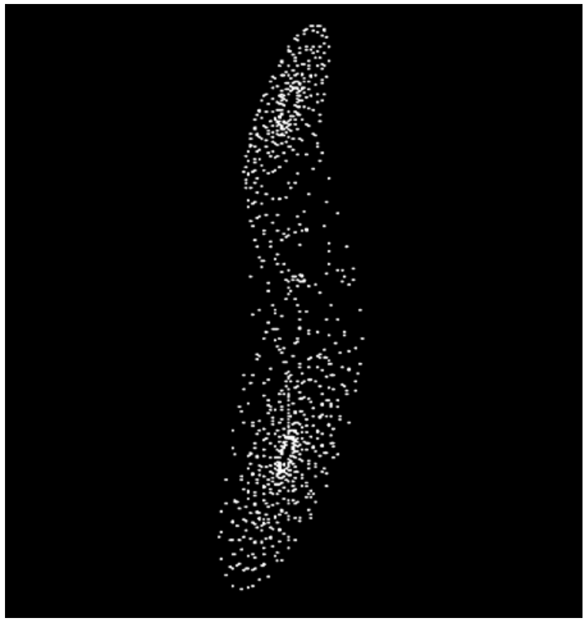


Figure 8.



2.4. Digital simulation of the Chua's oscillator

Extensive experiments in circuit control and interface design have been conducted using a digital simulation of the Chua's circuit. Control of the simulation is defined in terms of the parameters of the physical model of the unfolded Chua's circuit: R, C_1, C_2, L, R_0, and G_a, G_b, BP_1, and BP_2 of N_r (see equation 1). Digital simulation of signals from the Chua's circuit requires solving the associated ODE via numerical integration. The integration process approximates the continuous solutions by a succession of discrete points in fine quantization at a discrete time step, $\Delta\tau$. Since one of the characteristics of chaotic systems is sensitive dependence on initial conditions, the achievement of the maximum precision in calculating solutions is important in order to avoid errors that would effect solutions to diverge from intended solutions. For this purpose double-precision mathematics with 32 bit floating point and fourth-order Runge-Kutta method of integration (RK4) are employed. Using a fast graphics computer[3] we were able to achieve the digital simulation of Chua's circuit with RK4 integration computed with double-precision floating-points at a sampling rate of 32,000 Hz in real-time.

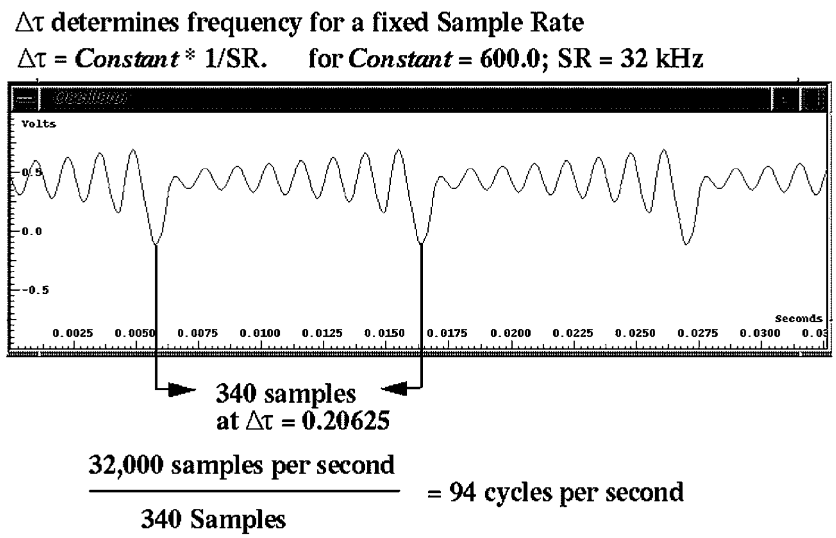
2.4.1. Timescaling the simulation for auditory signal production

ODE's are modeled in the computer as Ordinary Difference Equations, producing discretized signals with potentially smooth trajectories in phase space. For integration a discrete time step $\Delta\tau$ is assigned to *dt* (see Equation 1). $\Delta\tau$ determines the time interval at which successive samples are drawn from the continuous trajectory described by the ODE's. In order to use $\Delta\tau$ as an auditory timescale parameter, a *one to one* rendering ratio is selected. A one to one ratio indicates one sound sample is generated for every data sample. To apply timescaling we control the relationship between the time interval $\Delta\tau$ and the time interval of the sample rate (SR) of the D/A conversion. Simply put, samples are computed with respect to $\Delta\tau$ and displayed with respect to SR. Frequencies in the output signal can be raised or lowered by changing the ratio of $\Delta\tau$ to SR. We wish to keep SR constant in order to maintain the predictable frequency resolution of the output signal, therefor we specify the time interval $\Delta\tau$ in terms of SR. For a given sample rate, $\Delta\tau = C/\mathrm{SR}$, where C is a constant determining the ratio between $\Delta\tau$ and SR.

For an example let us take a periodic signal producing a well-tuned fundamental frequency. Figure 9 shows a period-8 attractor from the Chua's circuit simulation. We cannot tell the fundamental frequency of the period until we know two things: the sample rate at which it is reproduced, and the number of samples contained in one iteration of the period. For $C = 660.0$, $\Delta\tau = 20.625$ *ms*, producing a period-8 cycle every 340 samples. For SR = 32 kHz, this period-8 cycle comprised of 340 samples results in a frequency of 94.118 Hz. Increasing C the same trajectory is rendered with fewer samples, which signal converted at the same SR is perceived at a higher frequency. Similarly as C decreases the output frequency decreases. This timescaling method preserves a specified sample rate while optimizing frequency range.

[3] At the time of this writing the Silicon Graphics Indigo® series computer provides unique capabilities in a general-purpose computer for real-time D/A conversion of 16-bit audio signals at sampling rates up to 48 kHz, as well as support for fast rendering of 3D graphics.

Figure 9.

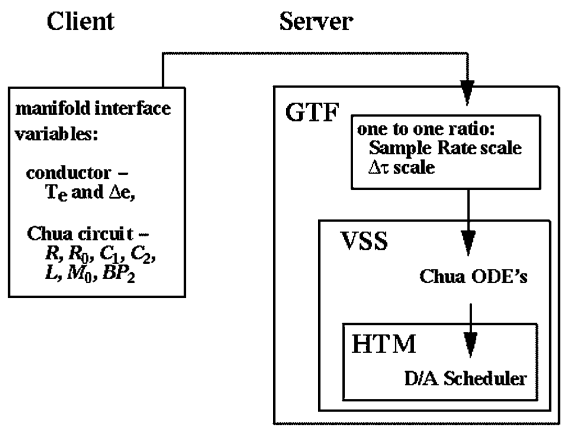


2.4.2. The unfolded Chua's circuit as a computational model for sound synthesis

Many of the experimental observations reported in this paper have been obtained from a numerical simulation of the Chua's oscillator, implemented in a digital computer such that it produces and converts digital signals to analog signals in real-time. The simulation of Chua's circuit is implemented in an interactive sound synthesis software environment. Figure 10 shows the software architecture.

"Real-time" refers to the operating time in performance context including a sound computation fast enough so that the delay due to the computational process between input and the results is virtually not perceptible. In performance practice which applies sound computation, the real-time execution capability is an important step to achieving auditory feedback. An auditory feedback nourishes performer's understanding how to control their own physical movements while interacting with their instruments in order to achieve intended sounds.

Figure 10.



The NCSA Sound Server [2] has been developed to support interactive, real-time sound synthesis on a standard Unix platform. The architecture of the Server has several layers: on the lowest level, a real-time scheduler for audio buffers (named HTM); above the scheduler, a selection of low-level synthesis algorithms (named vss); higher-level structures for managing the low-level synthesis routines (named GTF's: Group Transfer Functions), and a message protocol for controlling the Server from client programs. The Chua's circuit state equations (ODE's) are installed as one of the synthesis algorithms. Client programs include graphical interfaces for sending control signals in real-time to the parameters of the circuit simulation (see figures 16 and 17).

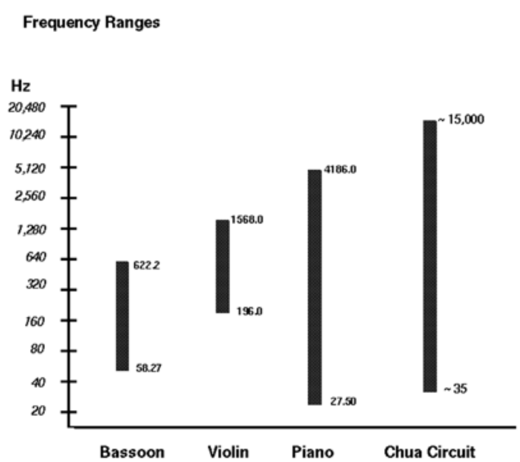
3. Acoustic Properties of Signals from Chua's Oscillator

The acoustic signals from Chua's circuit can be described in two ways; by using perceptual terminology such as *harmonicity*, and by generating descriptions based upon well-defined categories of attractors such as fixed point, limit cycles, intermittency and chaos. Attractors are patterns of oscillations. Given some initial conditions, transients are observed when the system goes through transitional states until it settles into an attractor. Various routes to chaos can be observed by applying parameter variation technique in order to study what parameters effect the system to display chaotic behaviors and what routes are taken to reach the chaotic state. In this section attractors and various routes to chaos will be discussed in terms of acoustic descriptions.

3.1. Frequency ranges and tuning

Many experimental circuit implementations operate at frequencies above the human hearing range. The audible frequency range is approximately 20 Hz to 20 kHz. The fundamental frequency of the circuit is determined by the energy-storage elements. Larger values of these elements result in a lower frequency for the circuit. In order to bring the frequency range of oscillations into the audible range for generating acoustic signals, we have specified 10 *nF* for C_2 and 57.4 *mH* for the inductor, *L* in Zhong's implementation of the circuit (see section 5.1). Figure 11 shows the frequency range of the specified circuit compared to other musical instruments.

Figure 11.



3.2. Tenor, harmonicity, spread

These terminologies are offered to generalize the descriptions of the observable qualities of the acoustic signals from Chua's oscillator. The attempt is to bind these qualitative descriptions to specific system properties and/or behaviors through music terminology. When the system dynamics reach a bifurcation state, *sub-tones* appear underneath the reference tone, sounding simultaneously. We will call this reference tone *tenor* in order to make an explicit distinction from the term *fundamental* above which upper partials are present in harmonic series of musical tones [10]. Further disctinction to be made between partials and sub-tones is that the partials are collections of pure tones, each partial consisting of a sine wave, while sub-tones from Chua's oscillator are not pure sine waves. Recall the frequency spectra in the FFT representation of period-one limit cycle in figure 1a.

The tenor and the series of sub-tones together constitute the content of frequency spectrum of limit cycles. The quality of harmonicity observed in limit cycles shares a common characteristic with musical signals from pitched instruments. Sub-tones emerge from the tenor in a bifurcation sequence, as the number of periods of the limit cycle increases. A tenor and the possible periodic solutions given the tenor, we will call a *tenor family*. Members of a tenor family are harmonically related to the frequency of a period-one limit cycle that is the tenor for that family. The frequency of the tenor is the same as the operating frequency of the circuit, therefore the frequency of a tenor varies as the ratio between L and C_1 varies. With the value of L fixed, one can achieve a variety of tenor frequencies by varying the value of C_1.

The degree of *harmonicity* reflects the distribution of energy in the frequency domain. When the energy in the power spectrum is distributed at regular intervals along the frequency axis the perceived quality of the tone is harmonic. The term, *pitch* refers to our perceptual response to frequency; often two or three distinct pitches can be heard in a complex harmonic tone. These pitches are in harmonic relation; sometimes the tenor is one of these pitches. When the energy is distributed at irregular intervals the tone is produced by a mixture of many periodic signals that rapidly alternate. The perceived quality of the tone is similar to band-pass filtered noise with formant characteristics.[4] Chaotic signals are noisy; the characteristics of their noises, however, are distinct from white noise which has a broad band spectrum. The spectrum of chaotic signals resembles 1/f energy distribution. In chaotic signals there seem to be pitch characteristics attributable to a centered energy concentration around specific frequency regions, independent of the amount of noise in the signals.

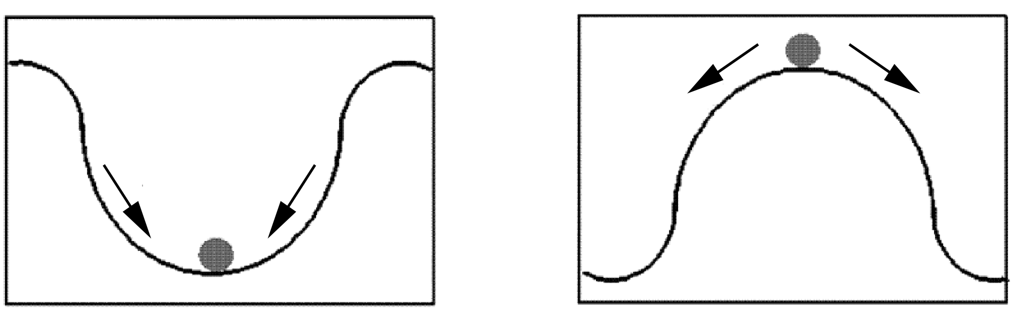
[4] Formants in natural sounds are created by resonant properties in vibrating bodies that tend to amplify fixed frequency regions in the upper partials of the complex tones produced by those vibrating bodies.

The term *spread* describes the frequency range of energy peaks in the signal. According to how finely the limit cycle is tuned around the frequency region the acoustical effect varies from a resonant harmonic quality to a pitched noise band. When multiple pitches or multiple bandlimited noise regions are present, *spread* describes the size of the perceived pitch interval encompassing these regions. The three characteristics, tenor, harmonicity, and spread, have been independently observed in signals from Chua's circuit. They appear to vary orthogonally as parameter variation is applied to the circuit.

3.3. Transients and Steady state behaviors

States of autonomous dynamical systems are characterized either by transient behavior or steady state behavior. Transient behavior occurs where an initial condition of the system locates a trajectory in a basin of attraction but not on an attractor. Steady-state behaviors are observed when the circuit is in an autonomous mode and transient behaviors have effectively decayed. These states may be categorized as either stable equilibrium fixed points, limit cycles, intermittency or chaos. Equilibrium points can be stable or unstable according to the parameters of the system. Figure 12 gives an intuitive picture for these two kinds of equilibria.

Figure 12.

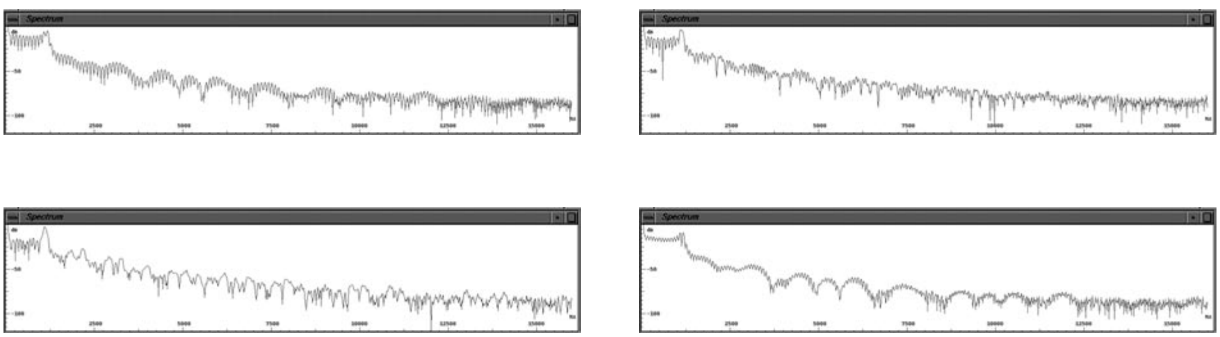


Stable Equilibrium Point *Unstable Equilibrium Point*

The acoustic characteristic of a fixed point is silence. The system is at rest so no sound energy passes through the transducer. When a system moves from a fixed point through a transient region towards an unstable equilibrium, sound emerges from the silence. When moving towards a stable equilibrium the silence takes over the sound. The opening phrases of the composition *anti-Odysseus, the irriversibility of time* [4] use a fixed point for phrase articulation by driving the system from a fixed point to an unstable condition and back to a rest state.

Limit cycles can also be stable or unstable. In experimental observation, limit cycles with long periods tend to be unstable. Other patterns in the neighborhood of the limit cycle can occur for nearby parameter values resulting in an alternation between limit cycles and irregular bursts. *Intermittency* refers to this phenomenon in which bursts of irregular energy or alternative periodic signals occur where the signal is periodic for longer periods of time. Intermittency is unpredictable and the degree of intermittency can be mild or strong in terms of the number of occurrences of the irregular bursts. Figure 13 shows several different samples from an intermittent signal; each sample was generated from the same parameter values.

Figure 13.



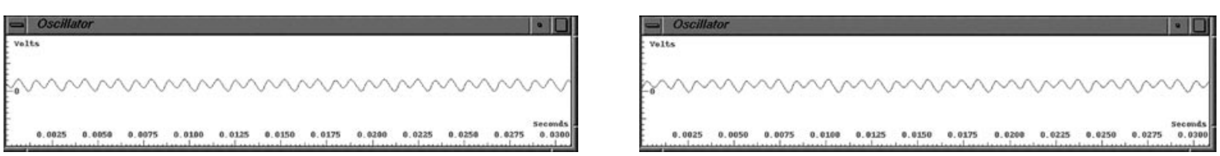
Chaotic attractors are steady state solutions in dynamical systems. They are comprised of many simple orbits that alternate in unpredictable succession. The orbits occupy bounded regions of phase space, often identifiable as chaotic neighborhoods of unstable limit cycles. The resulting trajectories produce complex auditory signals that exhibit noise characteristics and pitch characteristics embedded in the noise, easily distinguishable from the sound of broadband white noise. Chaotic attractors demonstrate a variety of acoustically differentiable timbres within noise. The difference in acoustical characteristics between the spiral and double-scroll attractors is present though not easy to detect. We hypothesize that small audible differences may be found to be related to the number of regions, D_1, D_0, and D_{-1} that the trajectory visits, and the varieties of transitions between regions.

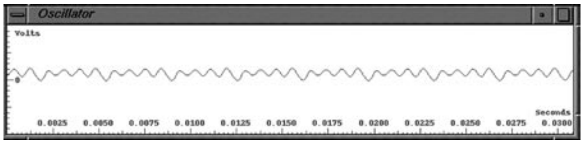
3.4. Routes to chaos

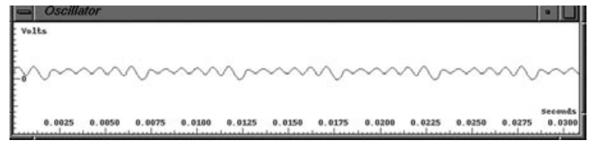
Beginning with the circuit at an unstable equilibrium point where no oscillation occurs, as the parameter *R* or C_1 varies with other parameters fixed, a bifurcation sequence is observed. A stable limit cycle of period-one emerges from the unstable equilibrium point, referred to as Hopf bifurcation, and the frequency of the oscillations of this limit cycle become the frequency of the tenor for the upcoming bifurcation sequence. With further decrease of the bifurcation parameter *R* or C_1, the stable limit cycle is destabilized and yields to another stable limit cycle of period two, of which the perceived frequency is an octave lower than the tenor, as the trajectory of the newly stabilized limit cycle takes approximately twice as long as the trajectory of the tenor to complete its orbit. The emergence of sub-tones in succession happens in integer ratio by multiplication of 2 from the preceding adjacent sub-tone, so the sequence proceeds from period 1 limit cycle to period 2 limit cycle, period 4, period 8, ..., period *n* , period *2n* limit cycle. This bifurcation sequence is called *period doubling*. In this route the bifurcation continues until the bifurcation point reaches a limit, an orbit of infinite period, beyond which chaos is observed. Through the period doubling sequence the tenor frequency is always present in weak amplitude contributing to the global tone percept, while the fundamental tone drops an octave at each bifurcation.

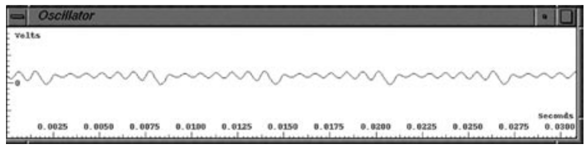
The bifurcation sequence with the emergence of a stable limit cycle followed by adding a period to the preceded limit cycle is called *period adding*. The newly stabilized limit cycle emerging from an *n*-period limit cycle consists of the period *n + 1*; the sequence proceeds by period 1, period 2, period 3 limit cycles, and so on (see figure 14). The perceived frequency contents of emerging sub-tones are reminiscent of the harmonic series in musical tones. The frequency ratio in harmonic series of well-tuned musical tones are 2:1, 3:2, 4:3, 5:3, 5:4, 6:5, 8:5, and so on; given the fundamental frequency, 66 Hz, the frequencies of partials are 132 Hz, 198 Hz, 264 Hz, 330 Hz, accordingly.

Figure 14.





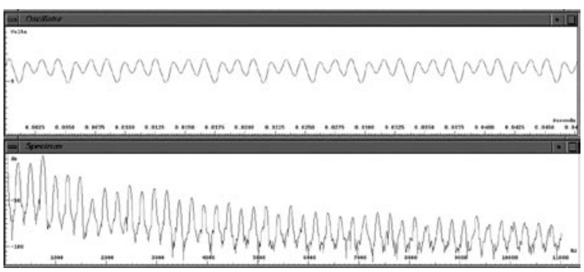




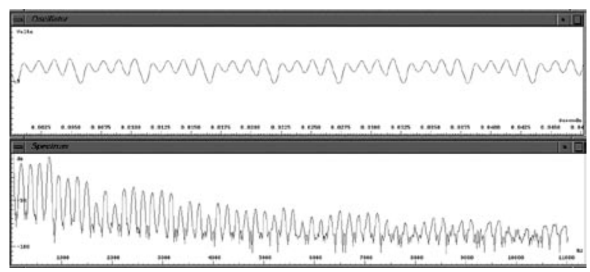
In the period adding sequence chaotic regions intervene between consecutive periods, as shown in figure 15. The waveforms of this intermittent region tend to consist of periods from neighboring periodic limit cycles.

Figure 15.

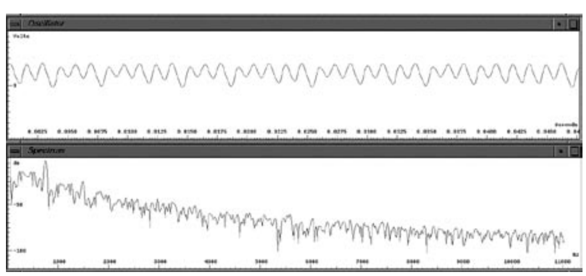
Period 3



Period 4



Region between Period 3 and Period 4



4. Interactive exploration of the circuit for generating musical signals

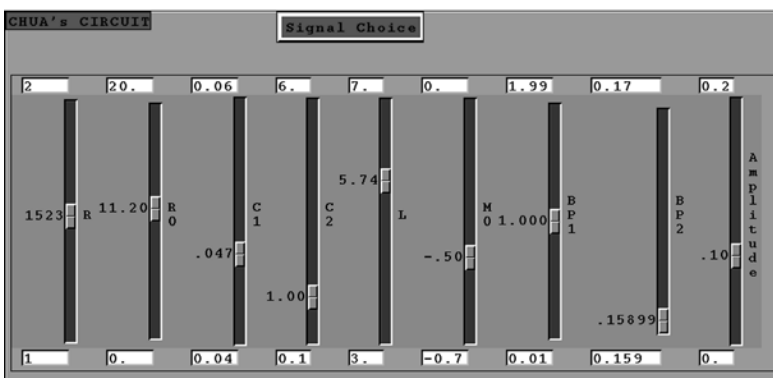
For immediate feedback from timescale exploration of a chaotic system, system parameters may be varied in real-time by an observer. An observer controlling the changes in the states of a system in real-time can generate a temporal observation. Gestures create auditory events, and we refer to the velocity of a gesture as *event time* T_e which expresses the rate a parameter value is varied under control of a gesture. We use Δe to refer to the change of T_e during an event, accounting for the speed variation of an observer's control which is seldom constant in a gesture. T_e and Δe are guided by auditory feedback as an observer listens for features and explores details. An understanding of the system dynamics comes from comparing the control rate T_e with the rate of change of the auditory signal. The observer develops an intuitive sense of T_e since she or he is generating gestures and receiving immediate auditory feedback responding to her or his actions. The observer can measure the robustness of the state changes caused by T_e by introducing fine variations in the control rate (Δe). Gestures may also be recorded and numerically measured in comparison to changes in the auditory signal.

4.1. Controlling the Chua's oscillator

The application of methods to influence the dynamics of chaotic systems is commonly referred to as *control of chaos*. Some authors prefer the term *taming chaos* in respect to conceptual issues addressing the presence of chaos and deviation from traditional control theory [17, 18]. One conceptual issue is to consider chaos a useful resource for extracting a variety of behaviors, by influencing a system through interplay with the chaos rather than "controlling" it.
The technique of parameter variation is among the simplest methods to influence chaotic signals. By changing the resistance, capacitance and inductance of the circuit components the state of the system will be changed according to its internal principles. Since the first confirmation of chaos in an experimental circuit it has often been observed that changing the values of circuit components can induce sequences of bifurcations and chaotic signals [32, 33]. These bifurcations can produce significant pitch, loudness and timbre variation. We have explored methods for generating specific sequences of auditory signals from Chua's circuit by applying transient control signals to multiple parameters, using time-specific, interactive, reproducible methods.

To study effects of parameter variation a circuit is needed with components capable of receiving control voltages from an external source. Using the simulation of the Chua's circuit, parameter variation was implemented as a test case and the knowledge obtained was later applied to the design and implementation of a voltage controlled analog circuit (see section 5.1). To facilitate explorations of the simulation a graphical interface has been adapted from [23] for real-time interaction, enabling the mouse to control graphical faders that represent the ranges of voltages of the simulated circuit components (see figure 16). These graphics programs are independent of the software simulation of the Chua's circuit. Control values defined using the graphical interface are sent to the sound server or to the analog circuit. To help articulate the control space of the Chua's circuit, upper and lower parameter boundaries may be specified as fader minima and maxima interactively using the mouse and keyboard. This provides scaling from a control space to the range of each component being controlled. Auditory feedback is often consulted for selecting these control boundaries. Graphical display of the output signal and the short time Fourier transform (STFT) [23] in real-time, enhances the understanding of the relationship between the parameter values and the output signal.

Figure 16.

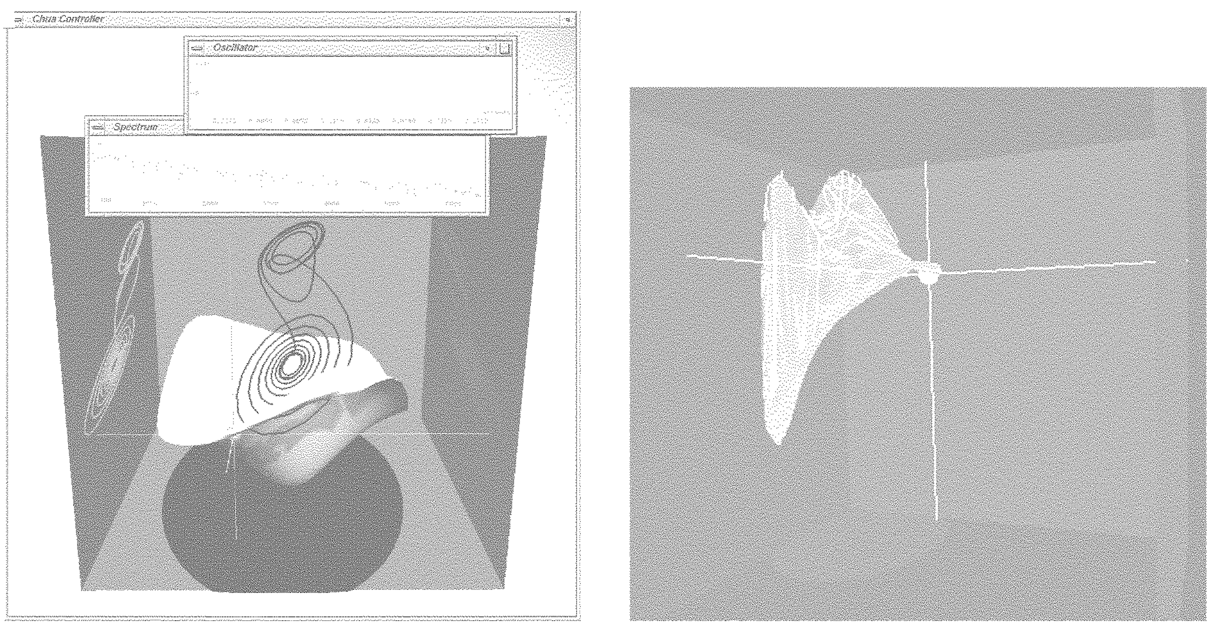


4.2. Capturing Gestures in a multi-dimensional control space

In order to nourish an auditory perception for exploring the parameter space of the oscillator an efficient and intuitive way of interacting with the system was designed. The simple analogy to such an interface for music instruments would be a bow for a violin, or keyboards on a piano to access the strings of the instruments to achieve necessary exitation energy for producing sounds. A system such as Chua's oscillator of which states can be defined and altered by a set of parameters would require the interface to offer ways of accessing these parameters. For digital environments the design of an interface suggests alternative approaches to physical interfaces due to different ranges of possibilities available in digital technology. At the same time the ability to capture traces of gestures left by explorers while they interact with the system was an important criteria for the design of the interface.

The *manifold interface* was developed to meet the need for an intuitive navigation of an arbitrary multi-dimensional control space (see figure 17). It was developed to access multiple parameters simultaneously for real-time parameter variation. Taking advantage of digital technology a graphical representation of parameter space helps to visually identify parameter regions of acoustically interesting signals. A user can vary the values of those parameters by varying the position of a cursor on the graphical representation of the parameter space. The virtual reality implementation of this interface to enhance navigation in immersive environments was presented in [1].

Figure 17.



The manifold interface is an exploration and composition tool embedded in a cube which defines projections of control values for *n* parameters; in the Chua's circuit case these are the six physical parameters {R, R_0, C_1, C_2, $BP+$ (also called BP_2) and m_0 (also called G_b)} [34]. Each axis of the cube provides a linear control dimension. One or more parameters may be controlled from each cube dimension. For each parameter the corresponding axis endpoints of the cube are assigned an upper and lower parameter value. In this way each cube defines a set of parameter range relations and ratios for parameter changes. Traversing the parameter space of the cube results in changes to the associated circuit parameters according to the functional definition of the cube axes.

To help specify regions in the cube where desired parameter combinations occur, control signals from the cube are obtained from graphical surfaces within the cube. A surface is defined by a continuous edge, a set of control points and spline functions[5] between control points. A single surface does not provide access to all possible parameter combinations in the cube, only to a subset selected by the investigator/composer. Decisions to position a surface in a cube are made by auditioning the sound signals generated at selected surface points. More than one surface may be defined within a cube. As the surface represents an arbitrary control space which may be considered an n-dimensional manifold, we refer to the graphical surface as a *manifold surface*.

Surfaces provide visual cues for potential trajectories in control space. Visual characteristics of a surface can become associated in a user's experience with specific sounds. Moving on a surface, the software allows a user to draw paths while listening to their acoustic result. Here drawing a path is equivalent to defining a trajectory in a three-dimensional control space. Paths record both the trajectory position and the speed of the drawing gesture. A user can "replay" a path and make adjustments to its rate and position. Recorded paths may be stored in computer memory and recalled. Paths may be combined to construct larger acoustic sequences. In the Chua's circuit case the rate of path traversal effects the resulting quality of the acoustical signal; the harmonic stability of a waveform can vary depending upon the speed with which a circuit

[5] Spline functions are a class of curves that are generated in relation to *n* control points. Spline curves originated from a graphic design technique for drawing smooth curves, by bending a flexible metal rule to fit with pressure against a number of pins that have been pushed into the drafting table. Some splines are affected only locally by control points; other splines are influenced by every control point at every point along the curve. Depending upon the particular spline, the curve may or may note required to pass through each of the control points.

signal converges onto an attractor, or transitions from one attractor to another, and the speed with which new circuit states are introduced by parameter variation.

4.3. Experimental reports of gesture-based explorations of Chua's circuit

This section presents descriptive reports from exploring Chua's oscillator with the manifold interface. The purpose of this report is to describe the experience of the interactive exploration with an auditory percept and introduce working vocabularies to attribute to the acoustic results achieved from the exploration.

4.3.1. Control Path

A *control path* is an *event* defined by a trajectory in control space and in time. Trajectories are defined by the gestures an observer makes using an external hardware interface such as *mouse*. In virtual reality implementation an event is controlled with 3-degrees of freedom and its trajectory is defined in 3-D according to the observer's physical movements with a hardware interface called a *wand*. In desk-top computer implementation, an event is controlled with 2-degrees of freedom and its trajectory is defined in a three coordinate system with one coordinate drawn as a function of the other two coordinates; the control path is drawn using a *mouse* and movement of the mouse is constrained in 2-D. Events are generated for making comparisons between multiple states in a simulation, or comparisons between parameter changes and state changes. Control paths are created during real-time observations and recorded in control space and in time.

A path encodes an observation itself as an event to the extent that the event time defined by T_e is recorded in the path and one can observe, when retrieving the path, T_e varies according to how fast or slow the observer was drawing the control path. This variation of T_e implies two things; (1) in terms of an observer's state of observation - when T_e slows down around the region of periodic attractor it implies the observer was searching for a stable boundary via her or his auditory feedback, (2) in terms of acoustic results - as T_e moves slowly the associated bifurcation sequence occurs accordingly reflected in acoustic results. We hear series of sounds changing the complexity and timbre according to the state changes. As an observer's gesture sweeps through the control path at fast speed the chaotic system does not have sufficient time to settle into any associated stable states; in the series of changes we will hear perceptually continuous sounds with unsettled transient quality.

The states that can be achieved depend in part upon the resolution of the Δ applied to parameters being varied. In the graphical interfaces Δ is defined in terms of a parameter range. For a given length of a control path, a small parameter range defines a fine Δ, a large parameter range defines a course Δ. The definition of an event by a gesture we feel is an important temporal unit for making observations. Listening while controlling T_e tends to encourage *searching* for states according to their associated acoustic results. We include Δe as a relevant variable because searching often employs Δe during an event to define regions of greater and lesser interest in a control path. In effect Δe introduces a temporal grammar to subdivide a gesture, to define meaningful paths within a continuous control space.

4.3.2. Spectral Focus

Spectral focus is a characteristic of auditory perception related to the complexity of an auditory signal. Spectral focus encompasses the frequency-domain characteristics *harmonicity* and *spread* (see section 3.2). Human ears are sensitive to energy distribution in the spectrum of signals such as can be seen in an STFT of a signal. Energy peaks contributes to perceived spectral focus and can be defined by their amplitude and their frequency distribution in the spectrum. *Searching* a control space involves acoustic orientation to the complexity and character of the signal. We have observed control paths slowing down as listeners focus on the details of regions according to acoustic criteria they have chosen. The criteria do not require external specification, they are arrived at by listening, encoded within a control path, and articulated by Δe. By re-examining recorded control paths and their effects on a simulation we become observers to another observer's interactions with the simulation. And our experience of time measure applied to the simulation involves both the signals from the simulation and the temporal grammar of the control path event.

4.3.3. Timbre Rhythm

Timbre rhythm is produced by an interaction between the perceived continuity of an auditory signal and discontinuities within the signal. Bifurcations are responsible for timbre rhythm, generating discontinuities that are rendered into a continuous auditory signal resulting in ambiguous temporal characteristics. This ambiguity produces a sound having a rhythmic presentation which is also a timbral presentation. Irregularity of discontinuities creates rhythms; when rhythms occur in sufficiently rapid succession they create timbre. The timbre results from fine details of the sounds in which the rhythmic characteristics are migrated into the micro-structure of the sounds. When there is an ambiguity due to a temporal unit which is neither long enough to classify its acoustic results in terms of rhythm, nor not short enough to attribute to the timbre exclusively, we achieve sounds with timbre preserving rythmic characteristics. Unlike steady-state or periodic waveforms, these timbres include a rhythmic roughness. The chaotic property known as intermittency also generates timbre rhythm (see section 3.3). Sparse intermittency involves a primary spectrum interrupted by bursts of alternative spectra, creating rhythmic patterns. As intermittency becomes less sparse the bursts merge and create a timbre. The signal no longer sounds like a rhythmic pattern, however imbedded in the timbre are rhythmic characteristics.

Our ability to observe system dynamics may be enhanced by results obtained with the ability to simultaneously vary multiple parameters of a Chua's oscillator. At the same time it is auditory feedback that provides the navigation ability in multi-dimensional control space, the "landmarks" of the state of the system, recorded in control space. This suggests an ability to interface with multiple parameters and control their real-time variation will increase our ability to explore the states of a simulation and to articulate structures implicit in those states. Results also suggest the implementation of auditory observation significantly enhances our ability to distinguish fine variations in attractors which cannot be observed otherwise in real-time.

5. Chua's Oscillator in a Concert Performance System

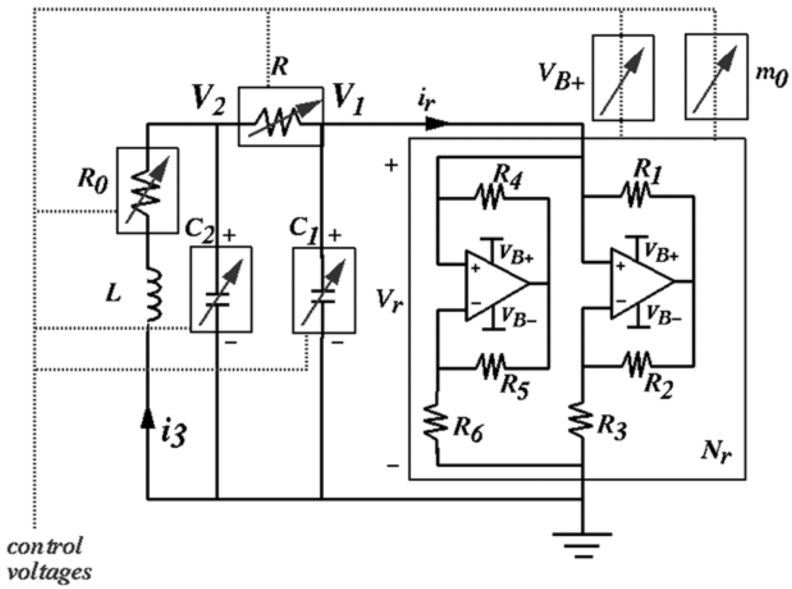
The digital environment for a performance system consists of the NCSA Sound Server real-time synthesis environment, the digital simulation of the Chua's oscillator, and the manifold interface. Special hardware for further signal processing are externally added to the system to create depth cues, reverberant spatial percepts and sound source localization in a stereo listening field. The addition of the analog voltage-controlled circuit enhances the performance system in terms of the liveliness and variety of the acoustic signal quality as well as contributing its identity as a solo or external partner of the simulated model of the circuit.

5.1. Implementation of a voltage-controllable Chua's oscillator for the *anti-Odysseus* project

To apply the parameter variation technique the circuit has to be implemented with components which are capable of receiving voltage control from an external source.[6] The implementation of the voltage controllable analog Chua's circuit is described in detail in [34]. The six parameters C_1, C_2, L, R, R_0, BP_2 and the G_a-G_b slope are implemented to be able to receive external voltages (see figure 18). The central component in the design of control circuitry is the analog multiplier, used to rescale the output of a variable capacitance diode, or "varactor" which originally has an output range too small for the control of the Chua's circuit. To rescale their voltages, each component is provided with a subcircuit consisting of an offset stage and an adder. The minimum value for the component is set by the offset voltage. Control voltages sent to the component are added to the basic offset value. The incoming voltages from the D/A converter vary between [0, 10] volts. This voltage range requires scaling to match the range that is appropriate for the element being controlled. This implementation includes an operational amplifier and two variable voltage-controlled resistors [30]. The resistors are used to specify the extremes of the desired voltage ranges for each parameter.

[6] The implementation of the voltage controllable analog circuit was designed and built by Q. G. Zhong, at UC Berkeley, and this device became an important instrument for acoustic exploration and realization of the composition, "anti-Odysseus (1993)."

Figure 18.



V_{B+} controls BP_2 of N_r; m_0 controls slope G_a of N_r

5.2. Protocol for connecting a digital interface to the analog circuit

To control the analog circuit, a graphical software interface requires a protocol for converting its digital values to analog control voltages. Two methods were considered: the use of a DSP board and the use of the MIDI[7] protocol. A DSP board[8] was identified for fast signal generation, however the DSP could not be controlled without going through proprietary software which was not designed to accept messages from other software applications such as the graphical interface. Therefore the MIDI protocol was selected for the prototype, with reservations toward the low resolution signal that MIDI supports, an integer range of [0, 127]. To control the analog circuit, software interface control signals were converted into MIDI controller messages, using MIDI libraries we authored for the computer. An off-the-shelf MIDI-to-CV converter[9] was used to translate the MIDI signals into analog control signals with the range of [0, 10]*V*. Seven cables carried signals to the analog circuit. Upon arriving at the circuit each signal is rescaled from [0, 10]*V* to a range specified by the composer, using the variable resistors described in section 5.1. Parameter range specification in the physical circuit

[7] The MIDI (Music Instrument Digital Interface) protocol was developed by the music technology industry as a standard serial communications protocol.

[8] The National Instruments Corporation NB-A0-6 Analog Output board for the Macintosh NuBus.

[9] The MIDI Retro/XLV manufactured by Clarity corporation.

is derived from the technique of parameter range specification in the manifold interface. For a discussion of problems arising from the low resolution of the MIDI protocol see [34].

5.3. Providing a signal processing context for performance

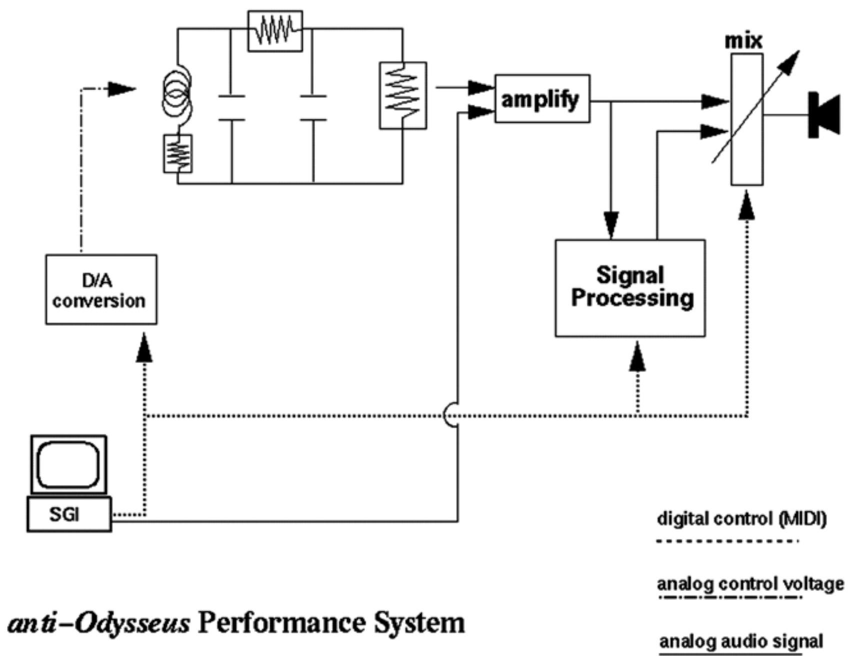
Signals from the Chua's circuit are treated in several composition contexts. Signals from the analog circuit are often accompanied by other signals which are produced by the simulated model of Chua's circuit. For timbre variation, signals are obtained from both C_1 and C_2 on the analog circuit; the signal from C_2 tends to be brighter and not as warm as that from C_1. All signals are generated as monaural signals, and duplicated for display in stereo. A computer-controlled analog audio signal mixer is used to combine and distribute the signals to the stereo field. Additional differentiation among signals is provided by controlling the loudness of each signal, and varying this in real-time. In order to simulate auditory distance cues, signals are further passed through computer-controlled filtering and time-delay signal processing units. Figure 19 shows the hardware configuration for a concert performance. Signals are generated in real-time during a live performance, so the additional processing of the signals by external devices is also required to be executed in real-time according to compositional and performance specification. Each of the signal processing devices in figure 19 is controlled by MIDI, and receives instructions from the performer through the computer.

In the future many of these functions could be included in a digital sound synthesis chip. See [9, 25] for examples of chip implementations of the Chua's circuit.

Conclusion and future projects

Complex musical signal generation from a multi-dimensional vector space opens a unique direction for composition and sound synthesis. Auditioning signals from complex systems enhances an intuitive understanding of the systems, and in turn, offers alternative ways of understanding those signals themselves. Signals may not be "musical," yet listeners may be drawn into their informative quality. Real-time and interactive sound synthesis environments such as HTM and the NCSA Sound Server have been a helpful platform to explore complex systems and their output signal responses to user interactions. The flexibility of the synthesis environment to host complex systems as computational models for interaction, is essential to the results presented in this paper. Chua's oscillator offered an excellent paradigm for exploring the output signals both from an analog circuit and a digital simulation; the output signals could be explored in digital simulation and tested in the analog circuit. This was possible since the circuit is mathematically proven so that one can accurately model the physical system in a computer.

Figure 19.



The ability to achieve compatible signals from two identities, one physical and analog, the other simulated and digital led to an interesting configuration of a real-time performance system. It also provided a philosophical ground for creating compositional problems to contextualize sonic space in time with signals from many like-identities. Differentiating control parameter space based upon auditory evaluations often preceded the defining of useful parameter spaces for musical signal generation.

The development of a graphical interface, the *manifold interface* for multi-dimensional control was an unique outcome of this project and it will continue in future projects, applied to other computational models which require high dimensional control. The ability to encode and retrieve the temporal aspect of gestures that define control paths will offer a playful exploratory platform for pre-compositional activities, thus enhancing the link between compositional activities and performance activities.

Acknowledgment

I am thankful to Robin Bargar who accompanied me from the beginning of this project to the present, for his mastery and generous hands, and to Camille Goudeseune for collaboration on the manifold implementation. I thank to Prof. Guo-Qun Zhong at U.C. Berkeley, and Nick Weber at CCSR for their encouragements, and their work was instrumental for experimentation and for executing a composition. Many delightful interactions with David Wessel and Adrian Freed at CNMAT, and Xavier Rodet at IRCAM brought me various insights while I was making directions for this project.

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